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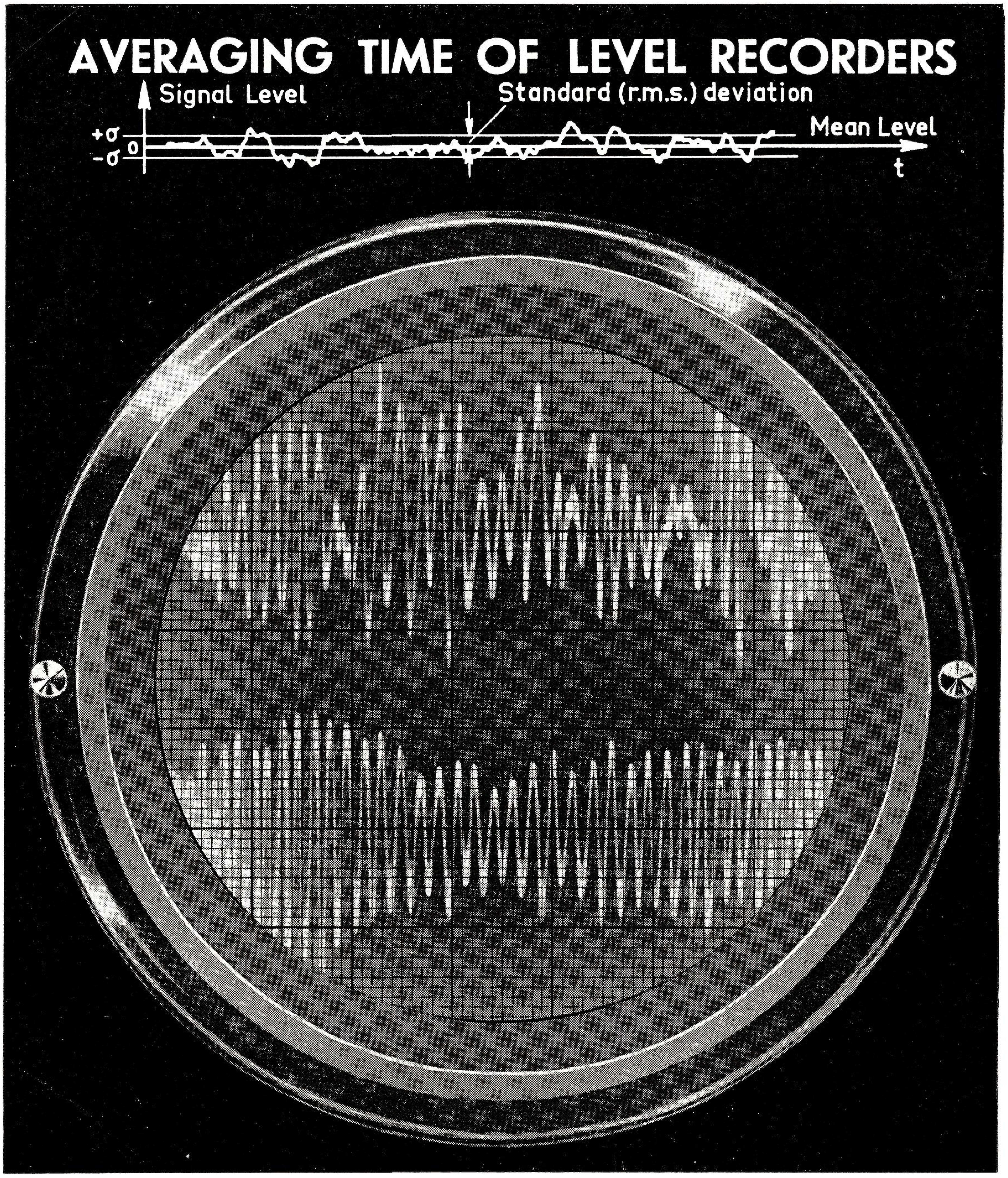

No. 1 1961

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TECHNICAL REVIEW

No. 1 – 1961

Effective Averaging Time of the Level Recorder Type 2305

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ABSTRACT

The theoretically true r.m.s. value of a statistically fluctuating signal can only be measured by averaging the squared signal over infinite time. This is, of course, not possible in practice, and modern measuring instruments contain an averaging circuit which allows the signal to be measured to a certain degree of accuracy. The accuracy of the averaging depends upon the effective averaging time of the circuit and the statistical properties of the signal to be measured. A number of formulae which, in most practical cases, describe the relationship between the measuring accuracy and the properties of the measuring circuit are given and applied to the averaging system in the Level Recorder Type 2305.

The dynamics of the Recorder writing system is thoroughly discussed and the frequency response characteristics measured. On the basis of these investigations curves of the effective averaging time of the Recorder are plotted. By means of a special measuring arrangement the results are verified experimentally and the various factors affecting the averaging time discussed.

SOMMAIRE

La détermination exacte de la valeur efficace d'un signal aléatoire stationnaire (bande étroite de bruit par exemple) devrait théoriquement se faire en prenant la moyenne du carré du signal pendant un temps d'intégration infini. En pratique la mesure se fait avec un temps d'intégration fini déterminé par les caractéristiques des circuits de l'appareil de mesure employé. L'erreur commise dépend de la longueur de ce temps d'intégration et des propriétés statistiques du signal fluctuant. Après quelques rappels théoriques des relations donnant le temps d'intégration de circuits simples, l'étude du cas de l'Enregistreur de Niveau Brüel & Kjær 2305 permet, à l'aide du réseau de courbes de réponse en fréquence du système d'écriture, d'obtenir le temps d'intégration de l'enregistreur en fonction de sa vitesse d'écriture. Le temps d'intégration peut également être mesuré à partir de la valeur efficace des fluctuations du stylet et les résultats expérimentaux sont comparés aux résultats théoriques. Enfin les variations du temps d'intégration du 2305 sous l'influence des divers autres facteurs sont discutées.

ZUSAMMENFASSUNG

Der theoretisch wahre Effektivwert eines statistisch schwankenden Eingangssignals kann nur gemessen werden, wenn man die Integration des gleichgerichteten Signals auf unendlich lange Zeit ausdehnt. Dies ist in der Praxis natürlich unmöglich, und moderne Messgeräte enthalten eine Integrationsschaltung, die den idealen Wert nur mit einer gewissen Genauigkeit zu messen gestattet. Die Genauigkeit der Mittelwertbildung hängt ab von der wirksamen Integrationszeit der Schaltung und den statistischen Kenndaten des zu messenden Signals. Eine Reihe von Formeln, die in den meisten praktischen Fällen die Beziehungen zwischen der Messgenauigkeit und den Eigenschaften der Messchaltung beschreiben, werden angegeben und auf die Integrationsschaltung des Pegelschreibers 2305 angewendet.

Das dynamische Verhalten des Schreibsystems wird sorgfältig besprochen, seine Frequenzcharakteristiken werden gemessen. Auf Grund dieser Untersuchungen wird die tatsächlich wirksame Integrationszeit des Schreibers in Kurven dargestellt. Mittels einer besonderen Messanordnung wurden diese Ergebnisse experimentell nachgeprüft und die verschiedenen Grössen mit Einfluss auf die Integrationszeit diskutiert.

When a statistically fluctuating signal, such as random noise is measured, the most significant amplitude characteristic to determine is the r.m.s. value of the signal because of the direct relationship between this value and the energy content of the signal in linear circuits. Another important characteristic is the frequency spectrum of the signal. The relationship between the r.m.s. value and the frequency contents of any signal is given by the integral: —

$$A^2_{\text{r.m.s.}} = \int_0^\infty w\,(f)\;df$$

where w(f) is the power spectrum density (volts² per c/s).
There are, however, other methods of determining the r.m.s. value of a complex signal. It can be determined as the standard deviation of the amplitude distribution curve, or it can be found from the time integral: —

$$A^2_{\text{r.m.s.}} = \lim_{T \to \infty} \frac{1}{T} \int_0^T a^2\,(t)\;dt$$

The most common method of measuring the r.m.s. value is the determination of the above time integral. However, the above definition involves the measurement over infinite time. This is, of course, not possible in practice, and an averaging time must then be used which defines the signal with an accuracy that is sufficient for the purpose of the particular experiment in question.

The averaging process can be made mathematically when an oscillographic time record of the signal is available. However, normally the signal is measured, rectified and averaged by means of a level indicating electronic instrument, thus saving the experimenter the time-consuming calculations involved in the evaluation of oscillographic records (samples of the signal), Fig. 1. This averaging process is catered for in the rectifier filtering circuit of the instrument. As the averaging action of such a filter is a continuous process, rather than a sampling process, the corresponding sampling time T of the filter should be defined. (The sampling time *T is not the same as the time normally designated as the time constant of the rectifier smoothing filter).*

It can, however, be defined statistically as *the averaging time of a rectifier circuit which will give the same level fluctuations in the measured r.m.s. value of the input signal as would the theoretical sampling time T defined by the formula given above.* This might require some further clarification, for example: —

If samples are taken of the signal to be measured, each sample having a duration of time T, and the r.m.s. value of various samples are calculated, the result will not be exactly the same for each sample. The calculated r.m.s. values will vary around a mean, the mean being the true r.m.s. value of the signal if an infinite averaging time is used, Fig. 2. The amplitudes of the variations around the mean

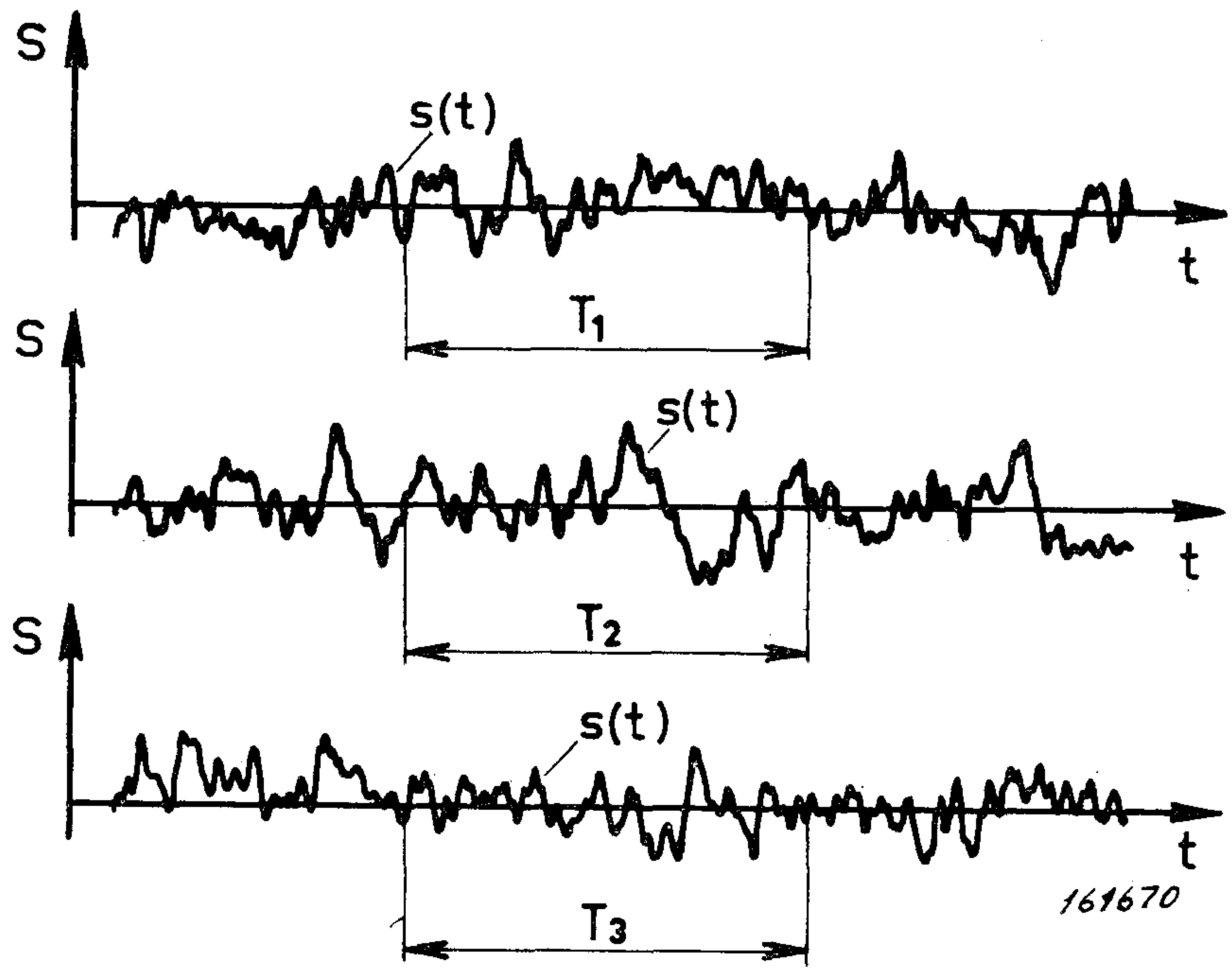

Fig. 1. Samples of a complex non-periodic signal.

value will depend upon the statistical properties of the signal as well as the sampling time T.

A practical statistical measure of the level fluctuations in the r.m.s. value of the signal is the "standard deviation" (σ), i.e. the r.m.s. value *of the level fluctuations.**)

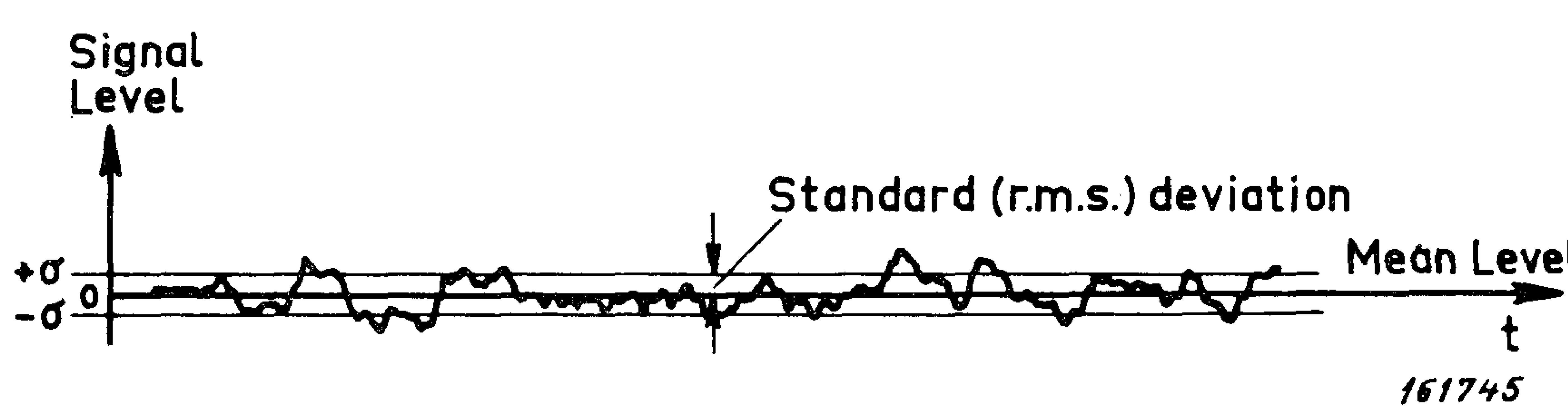

Fig. 2. Typical fluctuations of the r.m.s. values of a band of random noise around the true r.m.s. value. The standard (r.m.s.) deviation of the fluctuations are also indicated.

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*) The standard deviation of a continuous ergodic statistical process with zero as a mean is defined as

$$\sigma = \sqrt{\int_{-\infty}^{+\infty} x^2\, p(x)\, dx} = \lim_{T \to \infty} \sqrt{\frac{1}{T} \int_{0}^{T} x^2\, dt}$$

where x is the instantaneous amplitude value and p(x) is the amplitude density at x.

By determining the r.m.s. value of the fluctuations and comparing the result with known theoretical values calculated from samples of the same type of input signal the corresponding sampling time T of the rectifier filtering circuit can be found. For simple filtering circuits the effective sampling (averaging) time can be calculated mathematically from the circuit parameters. However, where more complicated filtering circuits are used the calculations become rather involved, and the easiest way of determining T may then be to measure the r.m.s. value of the fluctuations.

In the following a number of formulae will be given which will enable the determination of the effective averaging time for rectifier filtering circuits containing simple linear averaging networks. For the derivation of the formulae the reader is referred to the Appendix to this article and to the reference literature.

Let white noise be passed through an ideal band-pass filter, so that the output spectrum fulfils the requirements: —

$f < f_a$, output spectrum density $= 0$

$f_a < f < f_b$, output spectrum density $=$ constant

$f > f_b$, output spectrum density $= 0$

If this noise is squared and sampled over periods of time T, so that the product $B \times T >> 1$, where $B = f_b - f_a$, the r.m.s. (standard) deviation of the energy fluctuations is: —

$$\frac{\sigma}{A^2}_{\text{r. m. s.}} = \frac{1}{\sqrt{BT}} \qquad (1)$$

When the sampling process is substituted by the continuous averaging process taking place in an R-C averaging network the formula becomes: —

$$\frac{\sigma}{A^2}_{\text{r. m. s.}} = \sqrt{\frac{\pi f_o}{B}} = \frac{1}{\sqrt{2BRC}} \qquad (2)$$

By combining equation (1) with (2) it is seen that the effective sampling time of an R-C network is equal to twice the R-C time: —

$$T_{\text{sampling}} = 2\,R\,C = \frac{1}{\pi f_o} \qquad (3)$$

where f_o is the 3 db upper limiting frequency of the R-C network.

In some cases the averaging filter in the rectifier circuit is of the L-R-C type. The formula for the standard deviation of the energy fluctuations is then: —

$$\frac{\sigma}{A^2}_{\text{r. m. s.}} = \sqrt{\frac{\pi f_r Q}{B}} \qquad (4)$$

where f_r is the resonance frequency of the circuit and Q is the amplification of the circuit at this frequency.

The equivalent sampling time of this circuit is

$$T_{sampling} = \frac{1}{\pi f_r Q} \qquad (5)$$

Finally, if the L-R-C circuit is critically damped the corresponding formulae are: —

$$\frac{\sigma}{A^2_{r.m.s.}} = \sqrt{\frac{\pi f_c}{2B}} \quad (6) \qquad \text{and} \qquad T_{sampling} = \frac{2}{\pi f_c} \quad (7)$$

where f_c is the 6 db upper limiting frequency of the filter characteristic.

The formulae given above are derived on the basis of energy fluctuations of the noise and are therefore, strictly speaking, valid only for energy measurements. For small level fluctuations, however, the percentage energy fluctuations are approximately twice the percentage fluctuations in the r.m.s. (or arithmetic average) level of the signal:

$$\frac{\sigma}{A^2_{r.m.s.}} \approx 2\frac{\sigma'}{A_{r.m.s.}} \approx 2\frac{\sigma''}{A_{Ave}}$$

The main purpose of this article is to give an estimate of the averaging time of the smoothing circuits in the Level Recorder Type 2305, and with a view to this a number of measurements have been carried out. Firstly the frequency characteristics of the Recorder writing system were determined for various settings of the Recorder control knobs and the averaging time then calculated on the basis of the above formulae. Next the r.m.s. deviation of the pen fluctuations were measured and the result compared to the computed values.

In a previous article in the B & K Technical Review the r.m.s. rectifier and smoothing characteristics of the Recorder were thoroughly discussed and certain "standard" settings of the various control knobs suggested. It was found that to keep the servo of the Recorder stable only a limited range of "Writing Speed" settings should be used when the "Lower Limiting Frequency" knob was set to a value lower than 200 c/s.

Furthermore, it was found that the setting of the "Writing Speed" control knob greatly influences the smoothing of the rectified r.m.s. signal, and mention was also made of the influence of the "Potentiometer Range db" control upon the recording characteristics.

In the following a more detailed discussion on the frequency response characteristics of the writing system will be given.

Fig. 3 shows a block diagram of the Recorder, and it can be seen that a velocity dependent voltage is induced in the feedback coil of the writing system when the system is in motion. This voltage is subtracted from the "error" signal obtained from the "main" servo, the latter being amplitude limited to allow constant "error signal power" to be available over a wide range of error signal amplitudes. If there had been no limiters in the circuit, the velocity feedback would have given the writing system a frequency response corresponding to an analogous R-C network over the whole range of level variations, i.e. the frequency characteristic of the writing system would be independent of the maximum amplitude of the recording pen fluctuations. However, due to the limiters the frequency characteristic of the system changes as a function of the level variations.

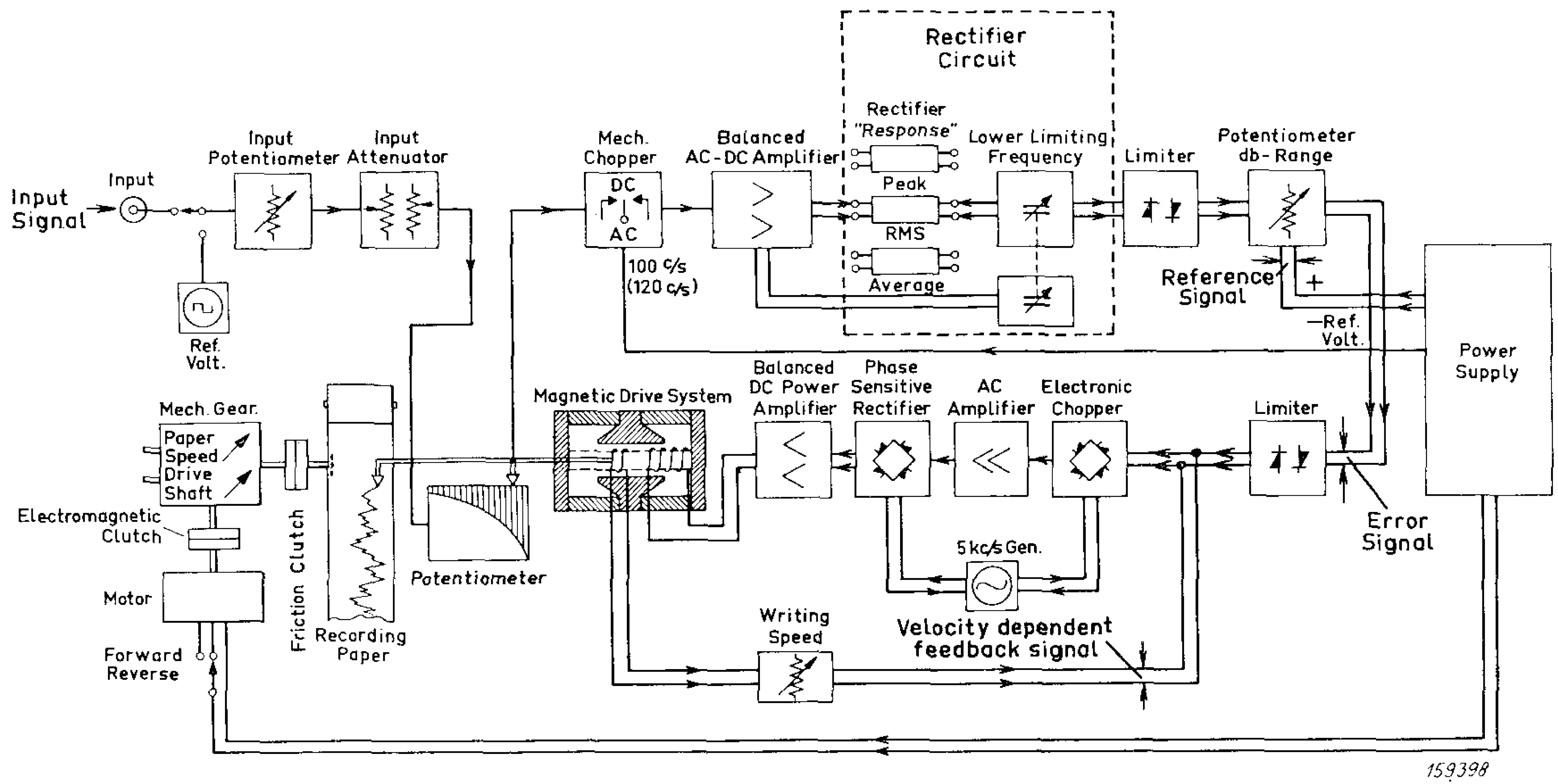


Fig. 3. Block diagram of the Level Recorder Type 2305.

Fig. 4 shows the arrangement used to measure the frequency response of the writing system. A 10 kc/s carrier frequency was modulated by a low frequency sinusoidal signal and applied to the Recorder input. The Recorder was supplied with a 10 db range potentiometer and switched for r.m.s. detection. The "Lower Limiting Frequency" switch was, during the experiments, kept in position "200 c/s".

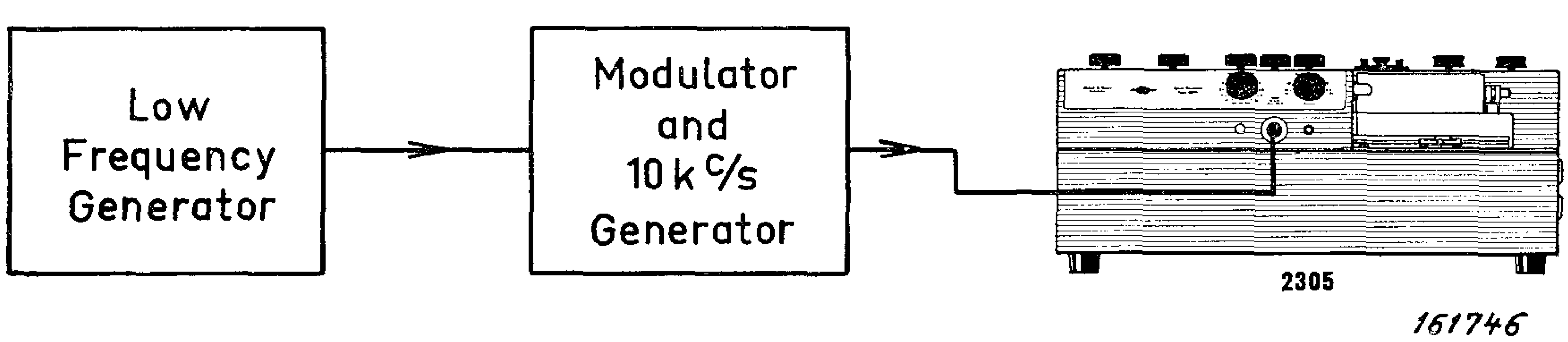


Fig. 4. Measuring arrangement used to determine the frequency response of the recorder writing system.

In Fig. 5 the frequency response of the writing system is plotted for different values of maximum amplitude (low frequency). The "Writing Speed" knob was in this case set to "100 mm/sec", and the "Potentiometer Range db" to "10". The amplitude was measured as the peak-to-peak deflection of the recording pen. Due to the limiters (Fig. 3) the expected frequency response should be as

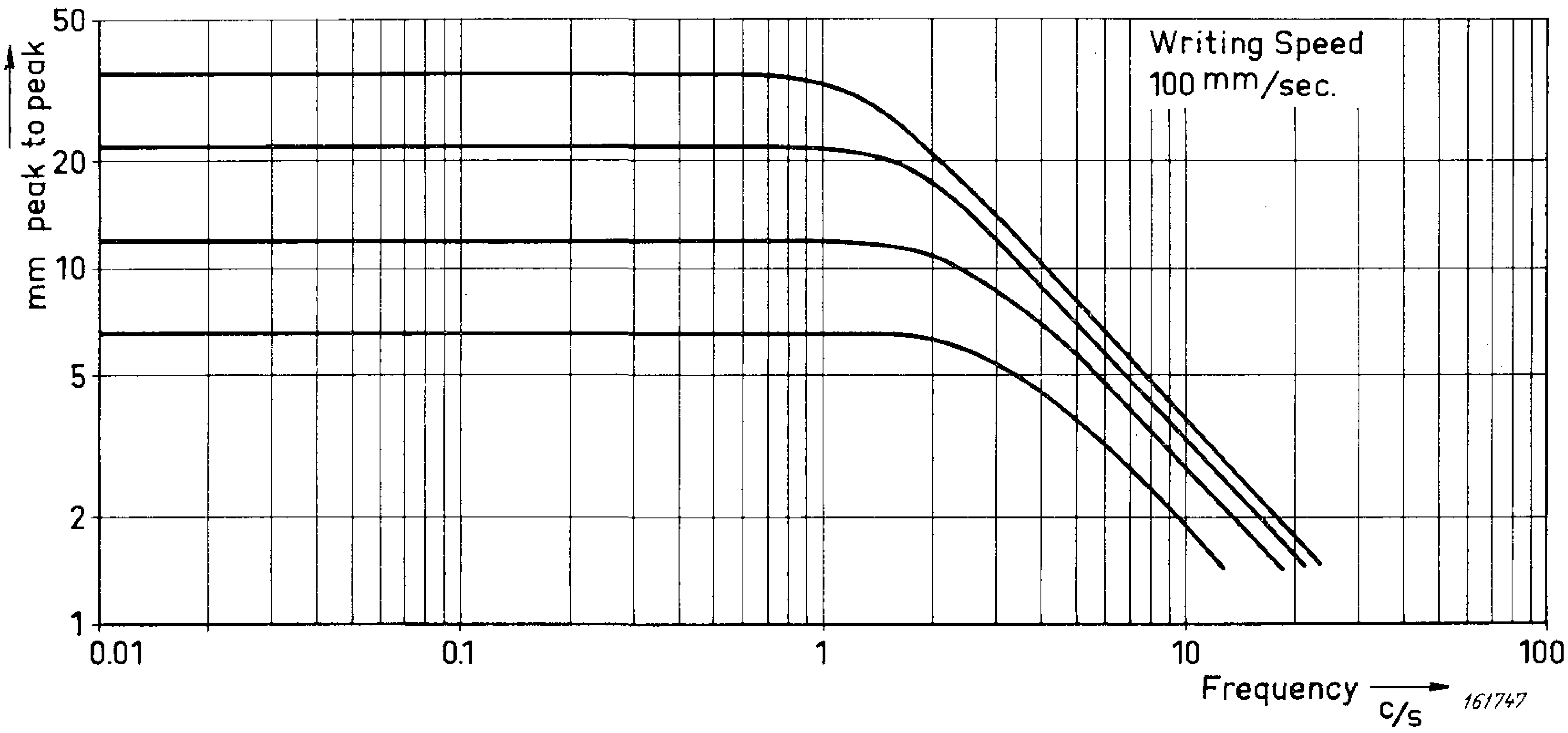


Fig. 5. Typical frequency response curves of the writing system for various maximum amplitudes. Measured by means of the arrangement shown in Fig. 4.

shown in Fig. 6. The 6 db/octave line (dotted) is determined by the writing speed. The reason why the experimental curves do not follow the theoretically expected curves is to be found in the resolving power of the Recorder, as this has a rather strong influence upon the writing characteristics. This can be seen from Fig. 7, where the response is plotted for one value of maximum pen deflection amplitude and various settings of the "Potentiometer Range db" control knob. Here also a writing speed of 100 mm/sec was used.

Fig. 8 shows the frequency response of the writing system for various writing speeds, and finally Fig. 9 shows the response both for various writing speeds and for various values of maximum pen deflection amplitudes. The setting of

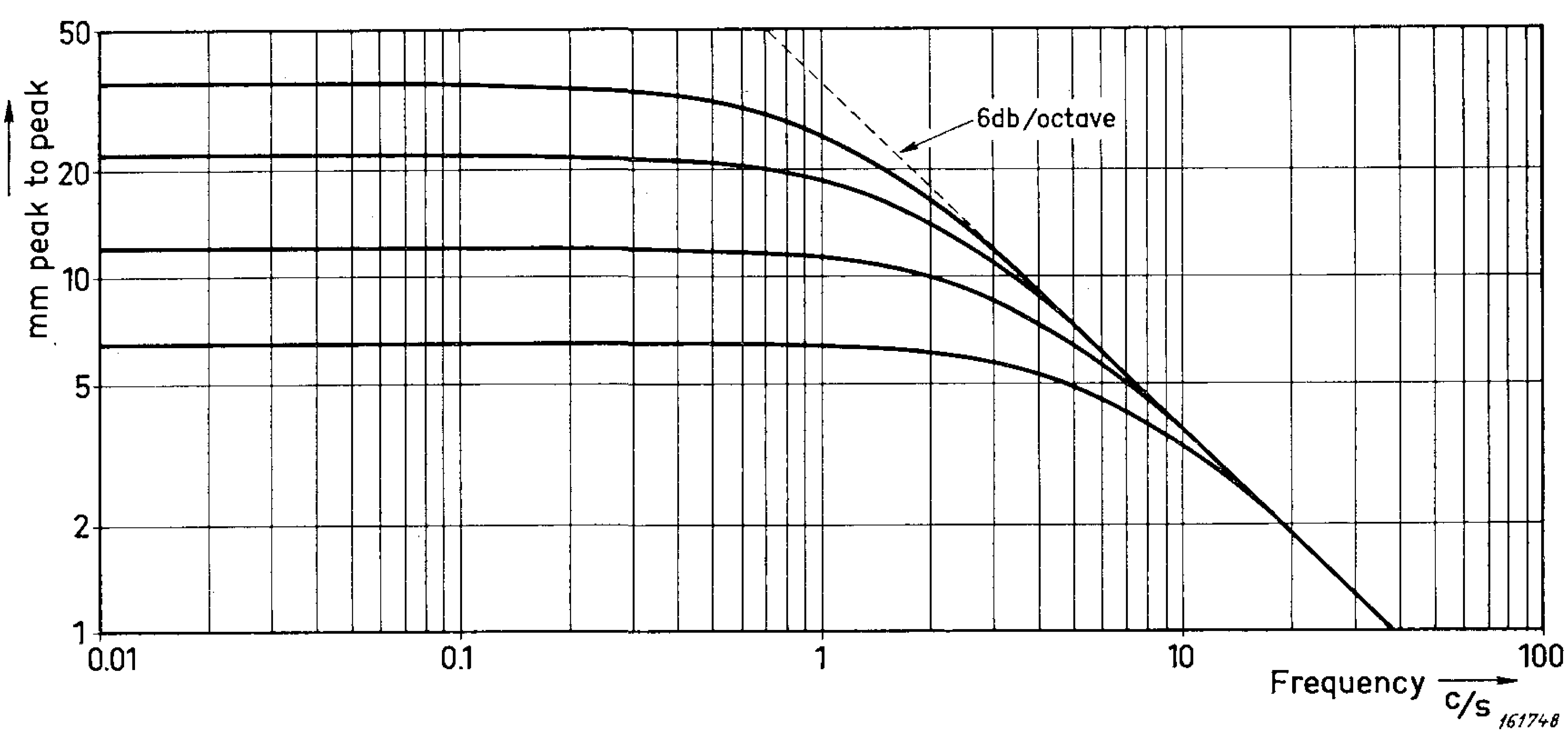


Fig. 6. Theoretical frequency response curves valid for a Recorder with infinite resolving power.

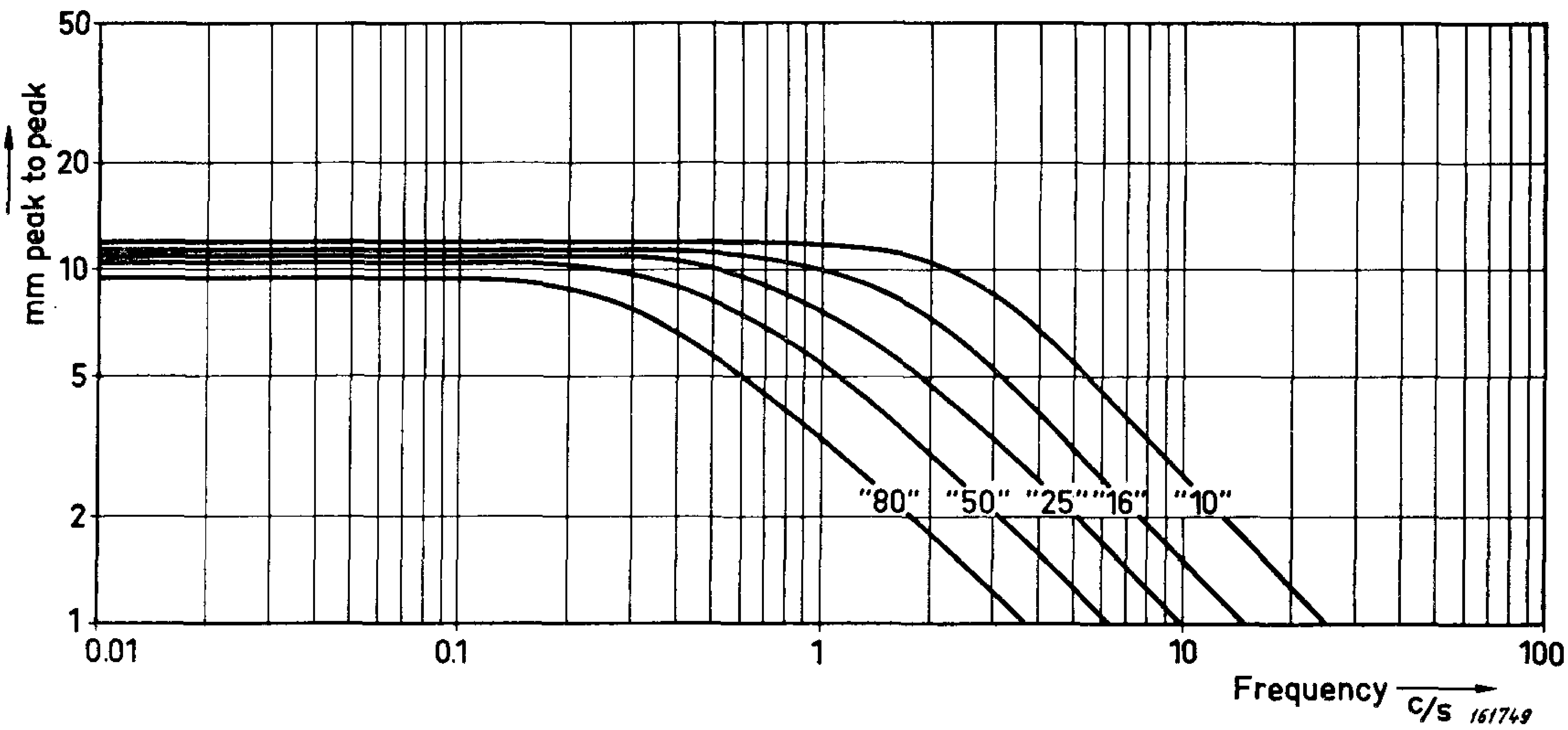


Fig. 7. Typical frequency response curves of the writing system for various settings of the "Potentiometer Range db" control knob (various resolving power), measured by means of the arrangement shown in Fig. 4.

the "Potentiometer Range db" switch was in all cases "10". In Fig. 9 the 3 db upper limiting frequency is marked by a dot on each frequency characteristic including the characteristics for settings of the "Writing Speed" control knob up to 100 mm/sec. For the W.S. = 500 mm/sec-characteristics the 6 db upper limiting frequency is marked on the curves, and for the W.S. = 1000 mm/sec-curves the resonance frequencies are identified by dots. Curves are drawn through the marked points showing how the limits approach a specific frequency value as the movements of the recording pen approach 0. For very small, or 0,

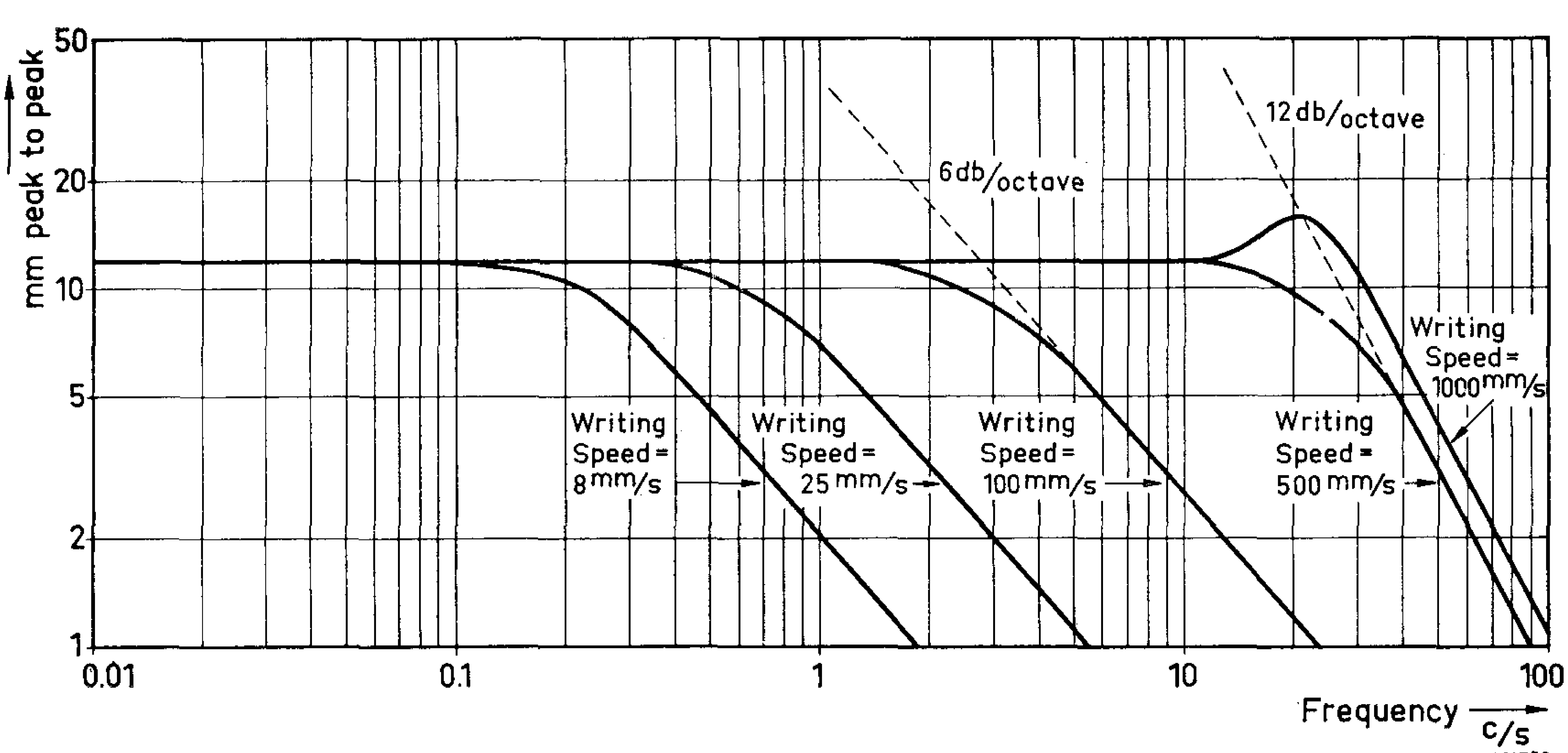


Fig. 8. Typical frequency response curves for "normal" setting of the Recorder control knob marked "Potentiometer Range db". The curves were measured for a fixed maximum amplitude and different writing speeds.

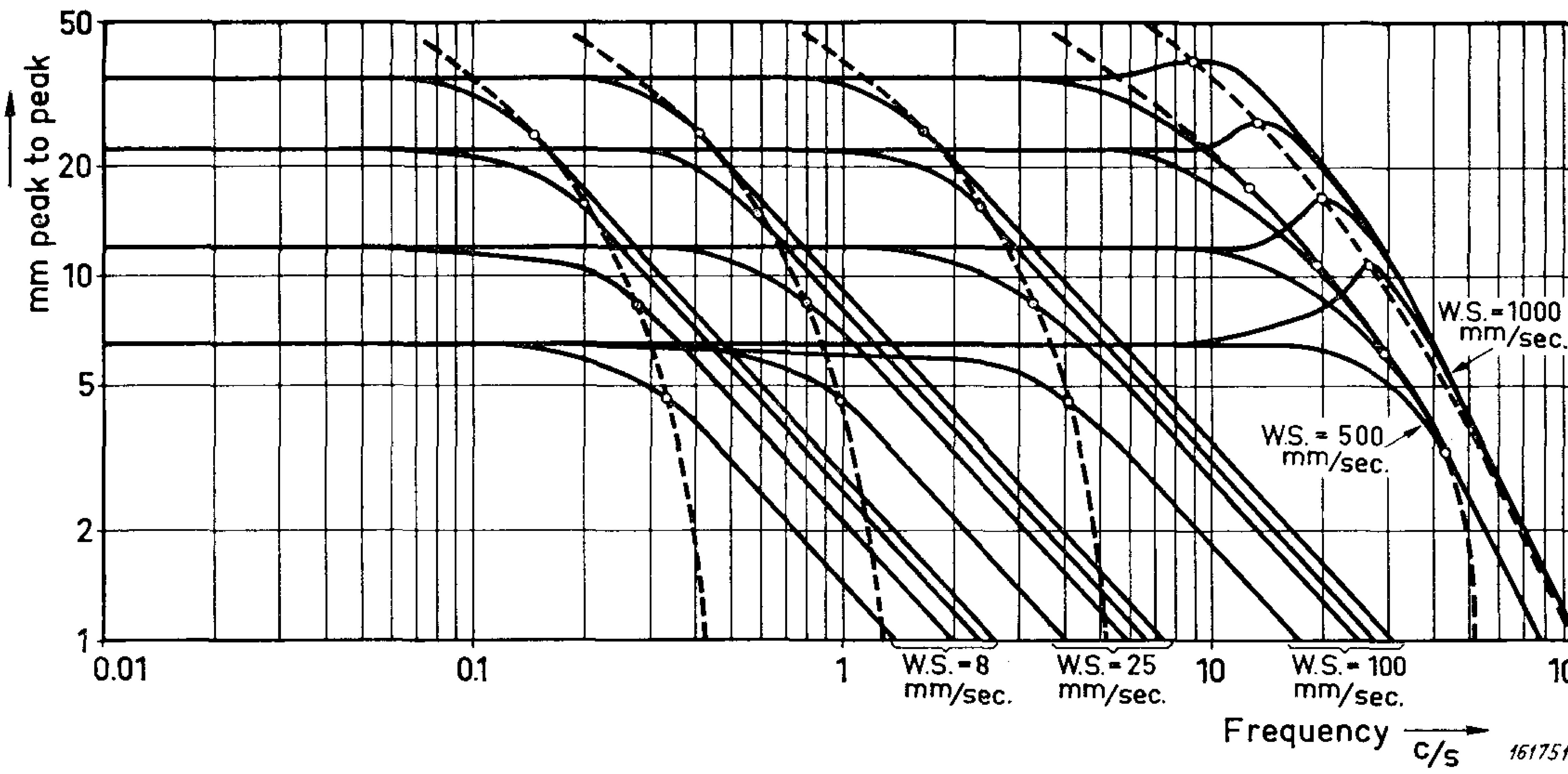


Fig. 9. Typical frequency response curves measured for different maximum amplitudes and writing speeds. The change in upper limiting frequency with maximum pen deflection amplitude is also shown (dashed curves).

pen fluctuations the writing system may thus be considered as a linear circuit. This effect can be made use of in determining the sampling (averaging) time of the Recorder. It was stated in the previously mentioned article in the B & K Technical Review that the averaging process of the Recorder mainly takes place in the writing system. For 0, or very small pen fluctuations and "standard" settings of the Recorder control knobs, the averaging (sampling) time can thus be calculated from the frequency response characteristics shown in Fig. 9 with the aid of the formula (3), (5) and (7) on p. 6 and 7.

Fig. 10 shows the result of these calculations. Formula (3) has been applied for the calculation of the averaging time for settings of the "Writing Speed" control knob below 100 mm/sec, as the writing system in this range can be considered as an analogous R-C type of network At W.S. = 500 mm/sec formula (7) was used. This formula was also applied for the determination of the averaging time at W.S. = 1000 mm/sec marked "Recorder stable" on the curve, while formula (5) was used for the dotted curve marked "Recorder unstable".

To attempt to verify these results experimentally the r.m.s. (standard) deviations of the pen fluctuations were measured when a narrow band of random noise was applied to the Recorder input. The sampling time T was then calculated from the formula (1) on p. 6. However, a number of factors which are difficult to determine accurately influence the result of these measurements, and the resulting accuracy can therefore be expected to be rather low.

Firstly, the formula (1) was developed on the basis of an ideal square-shaped band-pass filter, secondly, the fluctuations measured are somewhat between the fluctuations in the r.m.s. and the arithmetic average value of the noise band and finally the averaging circuit is not linear. The most critical factor is the non-linearity of the averaging circuit. The measuring arrangement used is shown

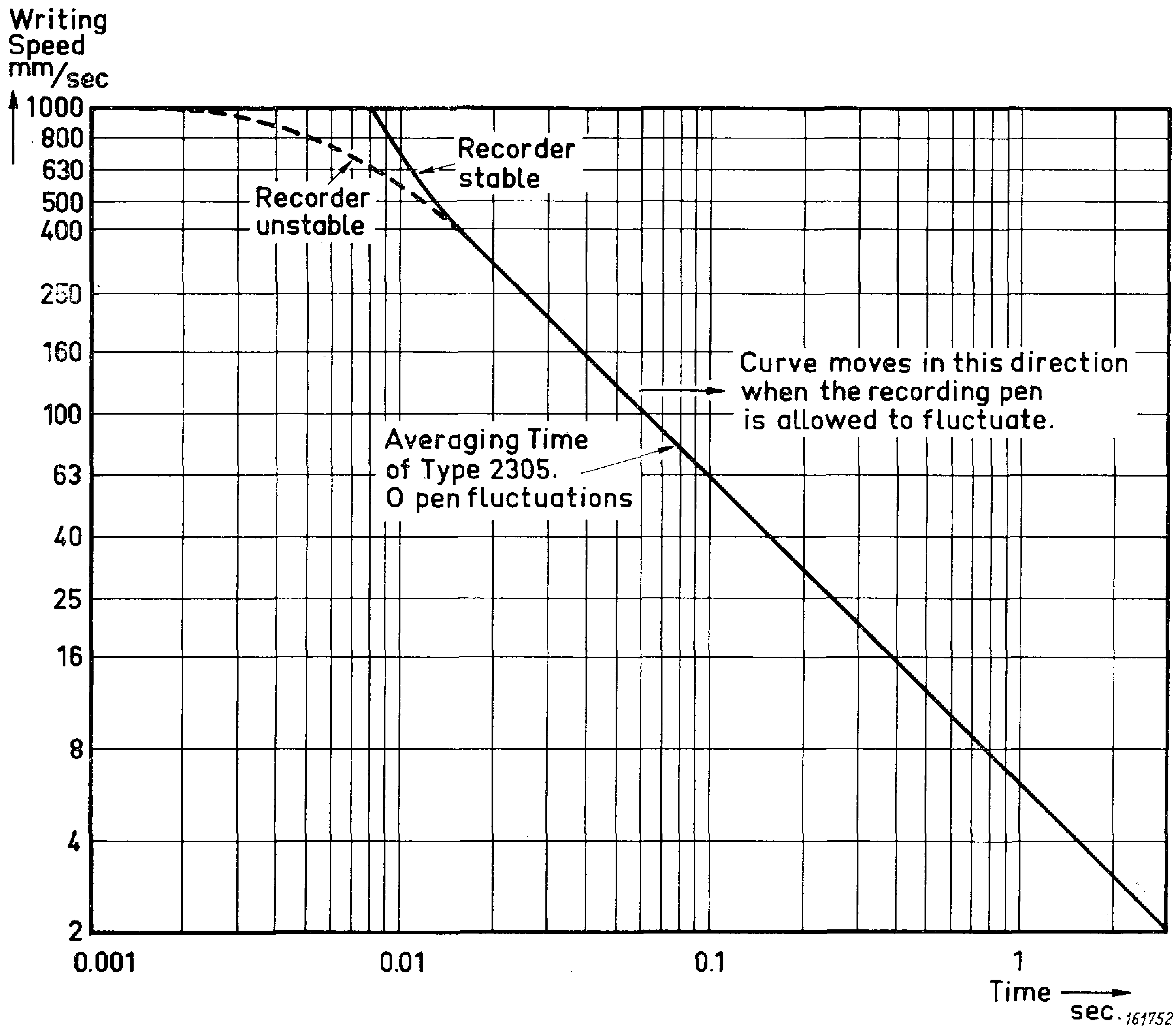


Fig. 10. Effective averaging time of the Level Recorder Type 2305 as a function of writing speed. The curve is valid for 0 pen fluctuations only and "normal" setting of the Recorder control knobs.

in Fig. 11. White random noise was passed through a one-third octave filter (Spectrometer Type 2111) and applied to the Recorder input. To enable the measurement of the r.m.s. deviation of the recording pen fluctuations the pen on the Recorder was substituted by a slider, and the recording paper by a "potentiometer", see Fig. 12. The mass of the slider was made approximately equal to the mass of the pen, and the friction between the slider and the "potentiometer" approximately equal to the friction between the pen and the paper. Because of the very low frequencies contained in the pen fluctuations at low writing speeds a special r.m.s. rectifier circuit with large time constants was used for the measurement of the standard deviation. A great number of measurements were taken on three recorders of the Type 2305. From the measurements it was found, as expected, that the setting of the "Potentiometer Range db" knob (the resolution of the Recorder), the setting of the "Lower Limiting Frequency" knob and the fluctuation amplitude chosen for the experiment greatly influence the measured result.

To keep the main servo in the Recorder stable the "Lower Limiting Frequency" knob was set according to the "rule" stated in the previously mentioned article in the B & K Technical Review. The "pen" fluctuation amplitudes were chosen so that the relative standard deviation of the energy fluctuations (twice that of the voltage fluctuations) was around 20% for all settings of the "Writing Speed" control. Fig. 13 shows the result of the measurements for the case when the "Potentiometer Range db" knob was set for minimum overshoot. By comparing this curve with the theoretically calculated curve for 0 pen fluctuations (dotted curve on Fig. 13) and the limiting response curves shown in Fig. 9 (dashed), it can be seen that good agreement exists between the two. Because the calculated curve was based on the assumption that a square-shaped band of noise was applied to the Recorder the difference between the frequency response of the 1/3 octave filters in the Spectrometer Type 2111 and the corresponding square-shaped band-pass filter was investigated. It was found that the difference in bandwidth would be of the order of 5 to 10%, which would introduce an error in the calculated σ-values of around 10%.

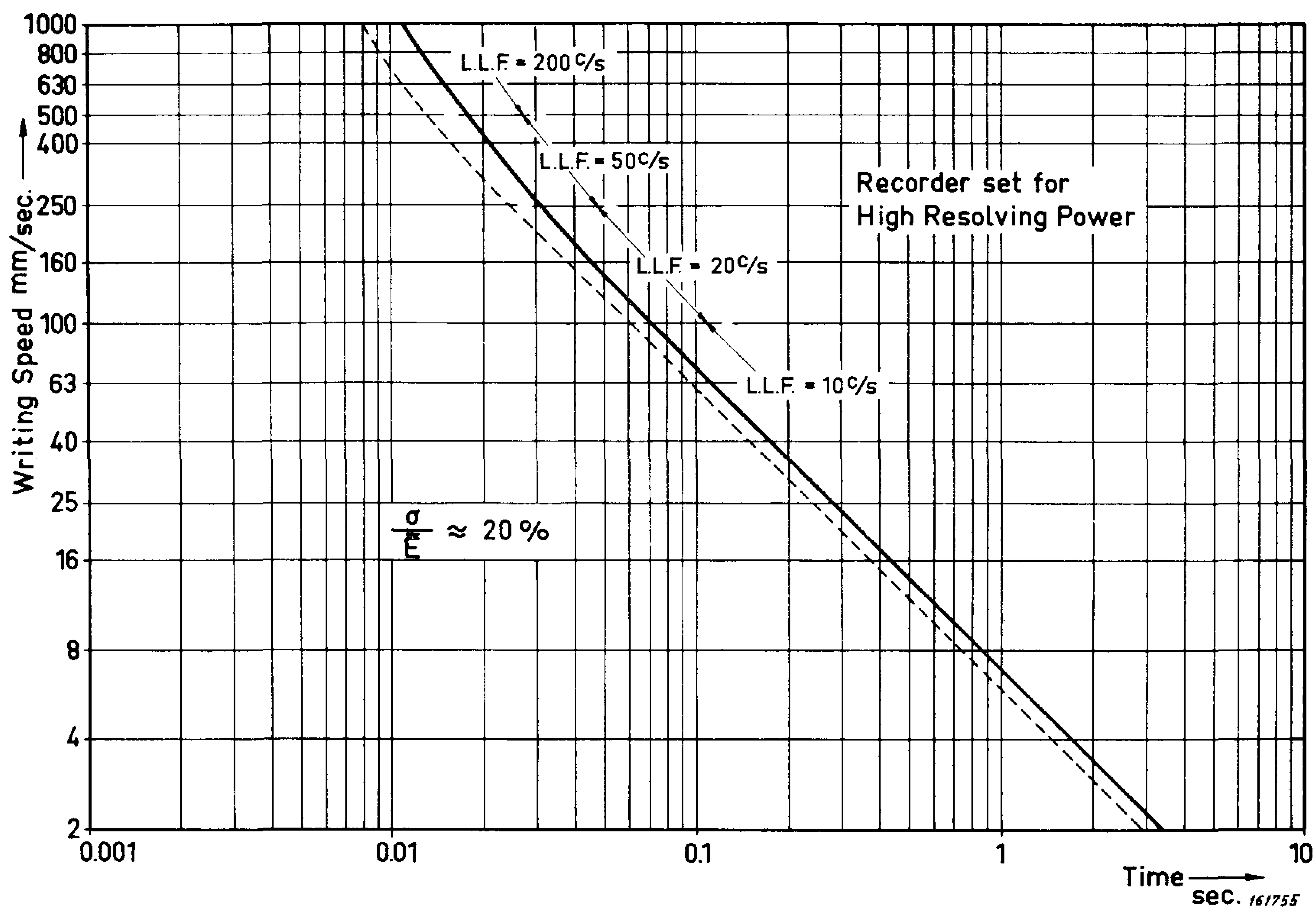


Fig. 13. Effective averaging time of the Level Recorder measured with the Recorder adjusted for high resolving power and 20% (r.m.s.) "energy" fluctuations.

To obtain an estimate of the influence of the amplitude of the recording pen fluctuations upon the averaging time the curve shown in Fig. 14 was measured. The curve is plotted for the case when the Recorder was set to high resolving

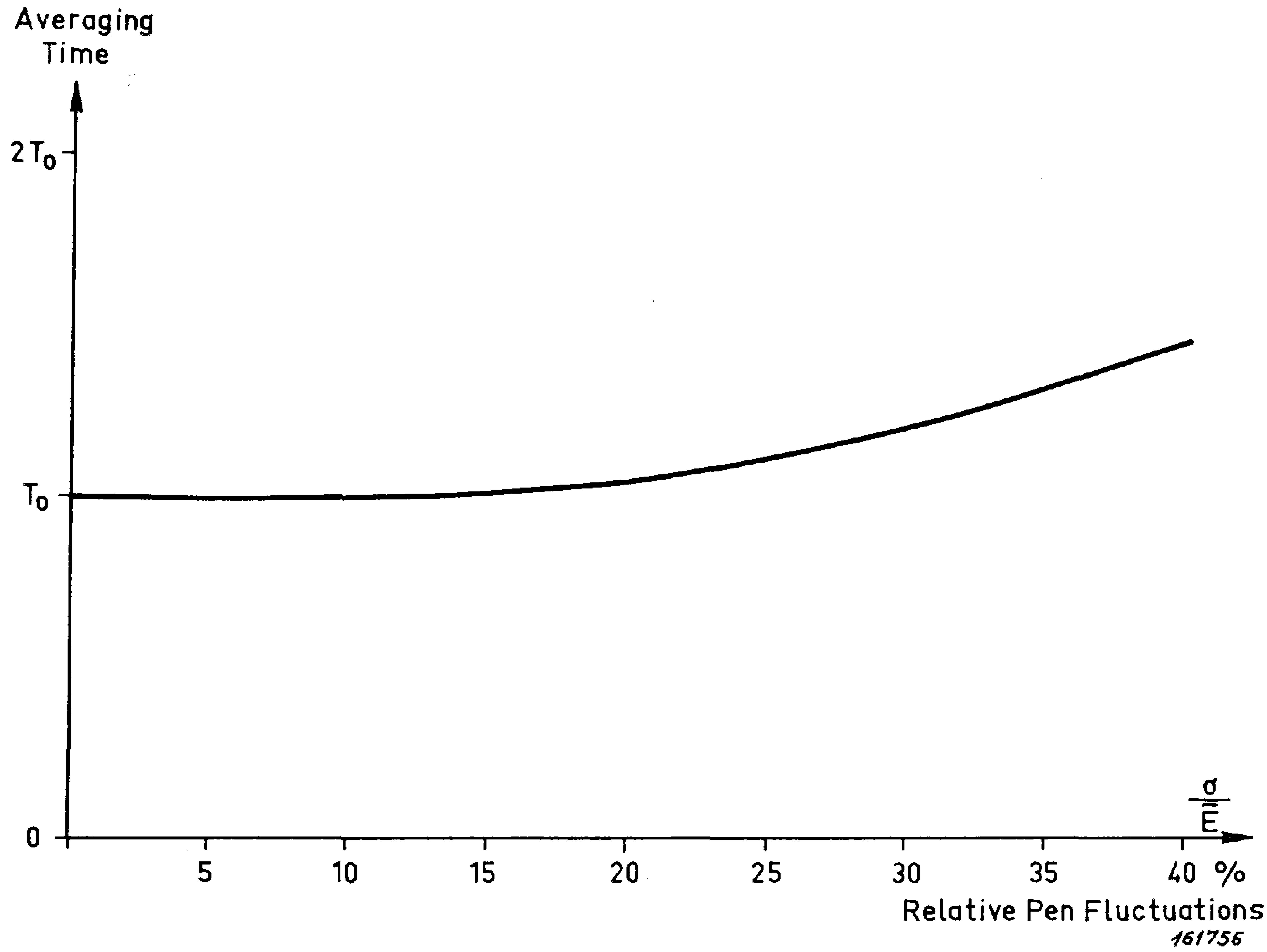


Fig. 14. Variation of effective averaging time with recording pen fluctuations. Curve measured with the Recorder adjusted for high resolving power.

power ("Potentiometer Range db" in position "10"). If a low resolving power is used, the curve will approach a straight line, i.e. the averaging time will be independent of the fluctuation amplitude. This is due to the gradually decreasing effect of the limiters, Fig. 3. At the same time the effective averaging time of the Recorder increases rapidly, which can be seen from the curve shown in Fig. 15, and also from the frequency characteristics shown in Fig. 7. The curve shown in Fig. 15 was measured with an r.m.s. deviation in the energy fluctuations of around 20%.

From the preceeding investigations it can be concluded that if the "rule of thumb" given in the B & K Technical Review No. 4—1960 with regard to the setting of the "Lower Limiting Frequency" switch is used (see also Fig. 13), the "Potentiometer Range db" switch is set for high resolving power (around position "10") and the amplitudes of the recording pen fluctuations are kept small, the effective averaging time of the Recorder will be of the order of magnitude given in Figs. 10 and 13. Furthermore, if recordings are made where the requirement for high resolving power is not strict, a considerably greater averaging time can be obtained by decreasing the resolution ("Potentiometer Range db" switch to positions around "80"). This may be utilized when spectrum analyses are carried out of low frequency random vibrations where long averaging

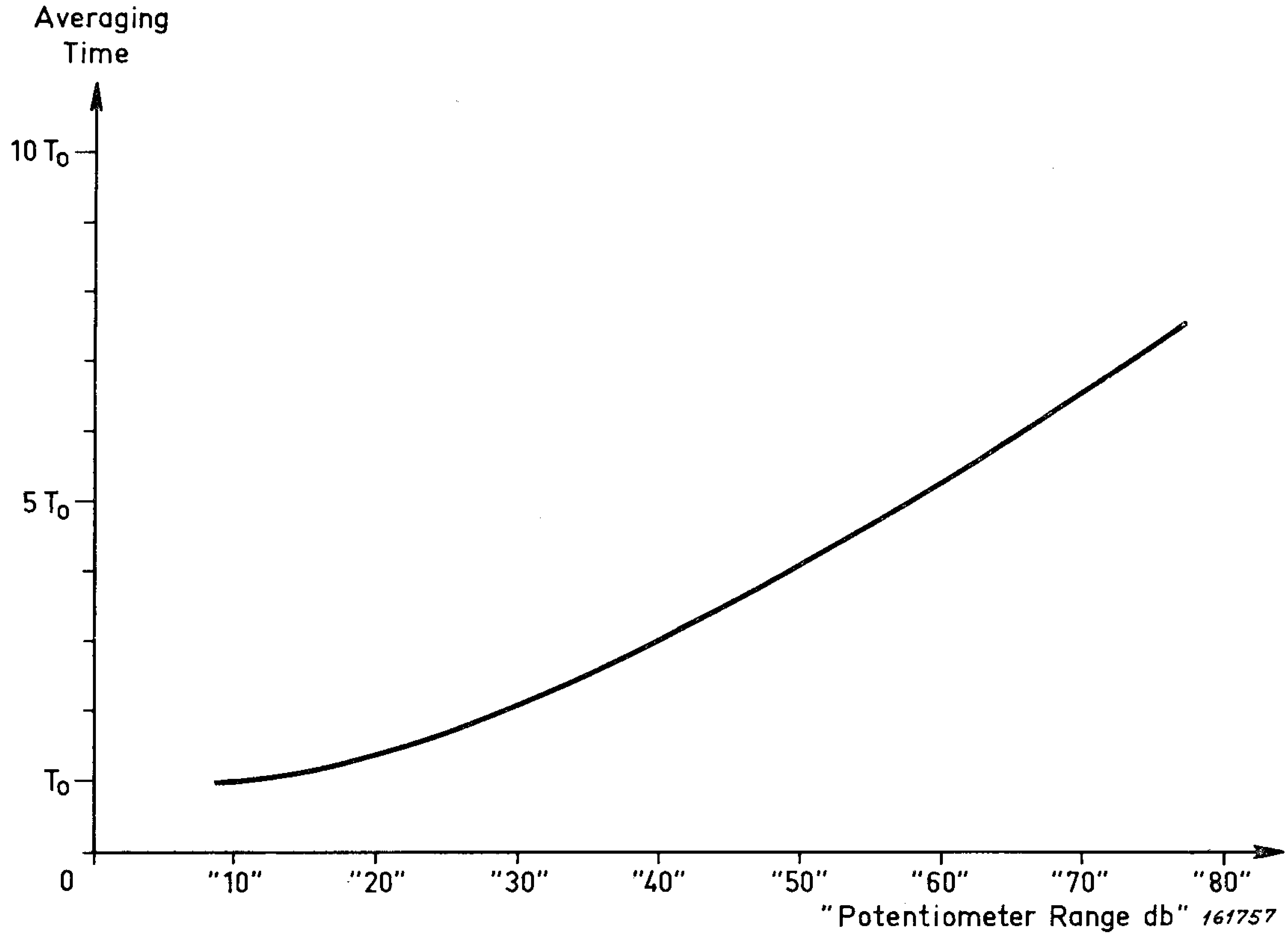


Fig. 15. Variation of effective averaging time with resolving power. X-axis notation refers to the setting of the Recorder control knob marked "Potentiometer Range db". The curve was measured with approximately 20 % (r.m.s.) "energy" fluctuations and then referred to O fluctuations (T_o).

times are required. It should, however, be borne in mind that the level indicated by the Recorder is then no more the true r.m.s. value of the signal but the mean r.m.s. value (see also the B & K Technical Review No. 4—1960).

APPENDIX

Derivation of the Standard (r.m.s) Deviation Formulae for the Energy Fluctuations of Narrow Band Noise.

In the following the derivation of the formulae used in the text for the standard deviation of the energy fluctuations of narrow band noise will be explained. Due to the rather complicated mathematics involved in the exact derivation only an explanation of one of the three known methods of computation is made. The result is then used to calculate the standard deviation of the energy fluctuations when these are averaged by means of different types of averaging filters.

The energy contents of a statistically fluctuating signal is, as long as linear circuits are considered, directly proportional to the square of the r.m.s. value of the signal: —

$$E \sim A^2_{\text{r.m.s.}} = \int_0^\infty w\,(f)\,d\,f$$

where w(f) is the power spectrum density of the signal at the frequency f.

If the signal is passed through a linear circuit with the frequency response G(f), such as an averaging filter, the energy at the output of this circuit is: —

$$E \sim A_o{}^2_{\text{r.m.s.}} = \int_0^\infty w_i\,(f)\,G^2\,(f)\,d\,f$$

where $w_i(f)$ is the power spectrum density of the input signal to the circuit at the frequency f.

The main problem in determining the standard deviation (r.m.s.-value) of the energy fluctuations of random noise is thus to compute the power spectrum of the *fluctuations*. When the power spectrum of the fluctuations is known at the input of the averaging filter it is a relatively simple matter to calculate the r.m.s. value of the fluctuations at the output of the filter (see formulae above).

To compute the power spectrum of the energy fluctuations of a band of noise the following considerations may be made: —

A white random noise signal, which is passed through a filter may be represented by an infinite number of sine-waves with relative amplitude values determined by the characteristics of the filter. The sine-waves combine in random phase. If the filtered noise signal is then passed through a squaring device (such as an r.m.s. rectifier) the output will be proportional to the energy content of the noise and will contain a dc component plus an infinite number of sine-waves with frequencies equal to the sum and difference frequencies of the input signal to the squaring circuit. (See also Fig. A. 1). The d.c. component is then equal

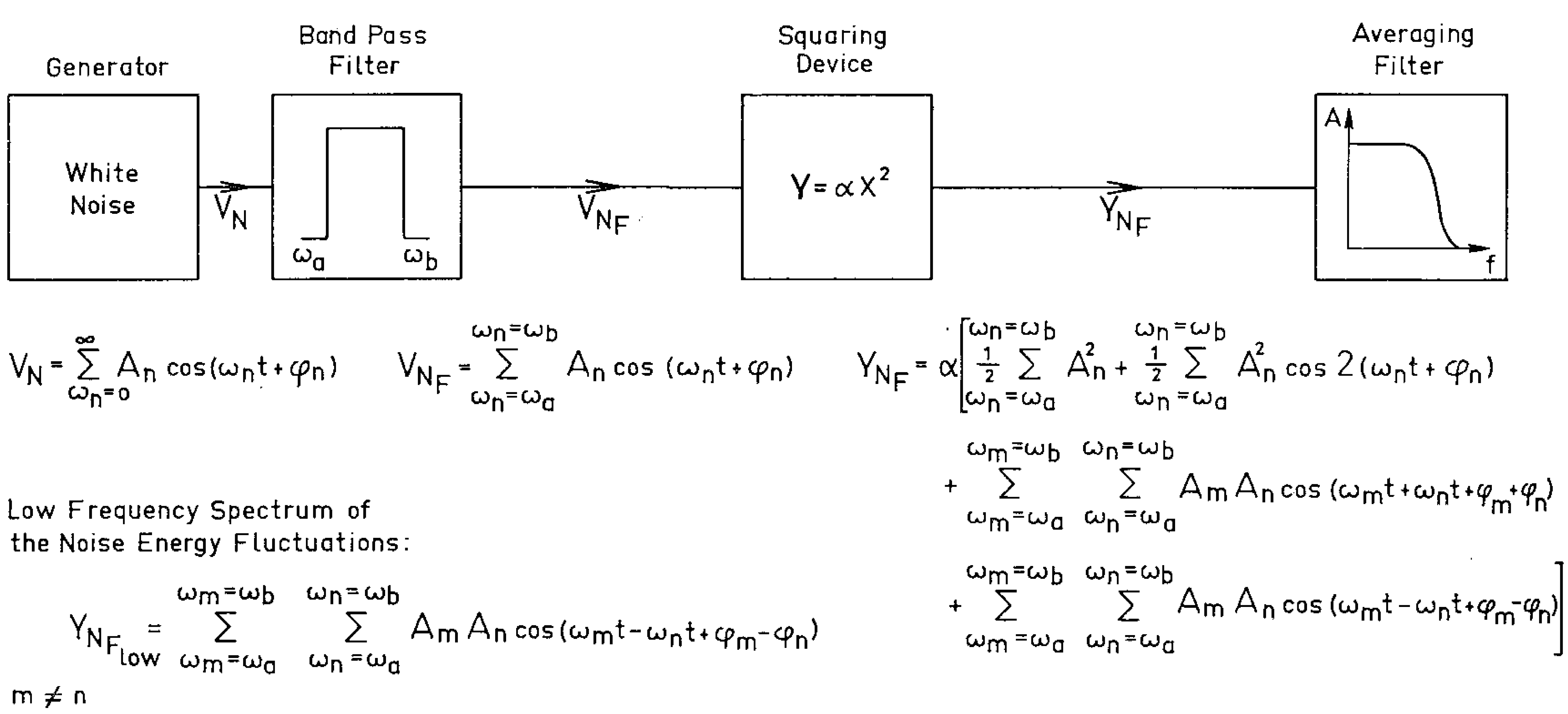


Fig. A.1. Block diagram of the filtering, squaring and averaging process of a power spectrum analyzer.

to the mean energy, while the a.c. signals cause the energy to fluctuate around this mean. When the signal is measured the energy is normally passed through an averaging circuit which has the characteristic of a low-pass filter. Only the frequency components with frequencies equal to the difference frequencies will thus contribute to some extent to the

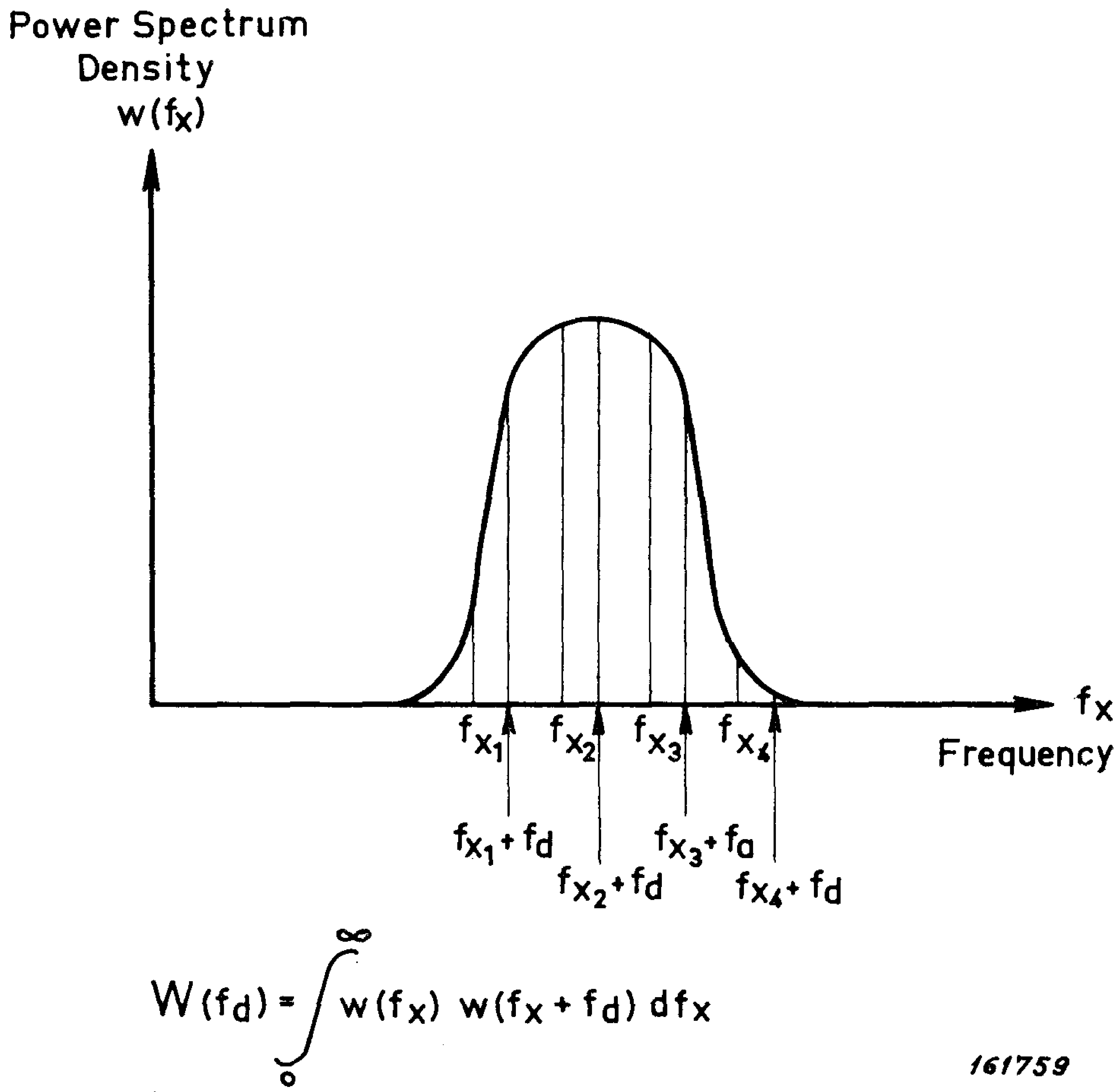


Fig. A.2. Sketch illustrating some of the possibilities of obtaining the difference frequency f_d by squaring a band of random noise. All signal components of frequency f_d add together in random phase to give the power spectrum density at f_d.

fluctuations at the ouput of the averaging filter. The amplitude values of each of these signals may, by simple trigonometric means, be shown to equal the product of the amplitudes of the original waves.*)

---

*) $A \cos \omega_m t \quad B \cos \omega_n t)^2 = A^2 \cos^2 \omega_m t + 2\,AB \cos \omega_m t \times \cos \omega_n t + B^2 \cos^2 \omega_n t$

$= \frac{A^2 + B^2}{2} + \frac{A^2}{2} \cos 2\,\omega_m t + \frac{B^2}{2} \cos 2\,\omega_n t + AB \cos (\omega_m t + \omega_n t) +$

$AB \cos (\omega_m - \omega_n) t.$

DC component: $\frac{A^2 + B^2}{2}$

Low frequency component: $AB \cos (\omega_m - \omega_n) t.$

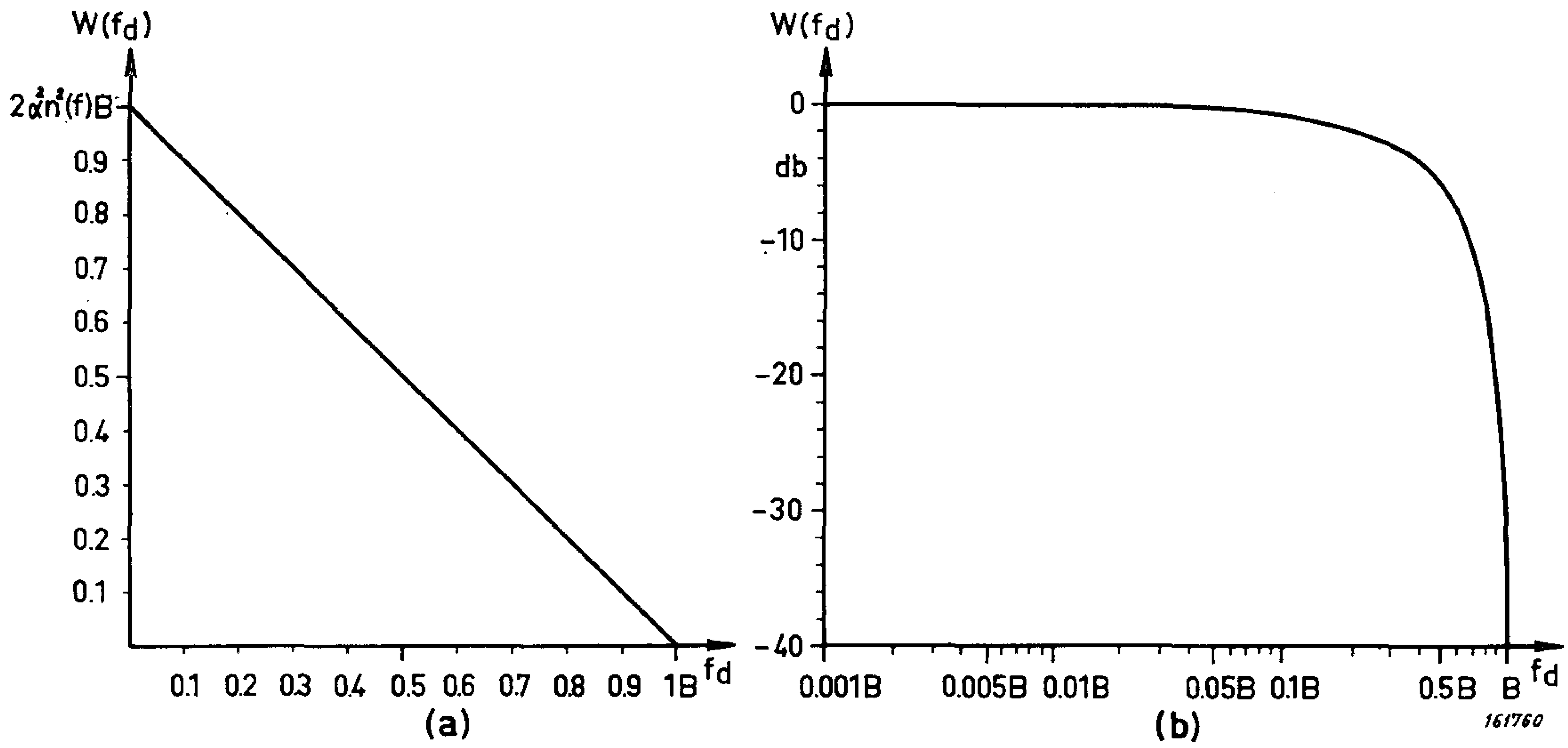


Fig. A.3. Low frequency spectrum of the energy fluctuations of filtered noice. The noise was here filtered through an ideal square-shaped band-pass filter of bandwidth B.

(a) Spectrum plotted to a linear scale.

(b) Spectrum plotted to a logarithmic scale.

Consider now an input signal to the squaring device which has a power spectrum of the type shown in Fig. A. 2. From the figure it can be seen that there are a great many possibilities of combining two frequencies f_m and f_n to give the difference frequency

$$f_d = / f_m - f_n / \text{ (or } / f_n - f_m /).$$

The power spectrum density of the low frequency part of the output signal from the squaring circuit at a particular frequency f_d is thus obtained by summing the products of the power spectrum densities of all the signals with frequencies f_m and f_n which give the same difference frequency f_d. This summation can be shown to equal the integral: —

$$W(f_d) = 2\alpha^2 \int_0^\infty w(f)\ w(f+f_d)\ df$$

where α is the "amplication factor" of the squaring circuit $(Y = \alpha\ x^2)$.
If white random noise is passed through a square-shaped narrow band filter and squared (see Fig. A. 1) the above integral can be written:

$$W(f_d) = 2\,\alpha^2 \int_{f_a}^{f_b - f_d} w^2\ (f)\ df = 2\ \alpha^2\ w^2\ (f)\ (B - f_d)$$

for $0 < f_d < B$.
Here $B = f_b - f_a$ = bandwidth of the square-shaped filter and w (f) is 0 when $f < f_a$ and $f > f_b$ and constant when $f_a < f < f_b$.
This spectrum is shown in Fig. A. 3. to a linear as well as to a logarithmic scale.
When the signal is passed through an averaging filter (low-pass filter) with a cut-off frequency $f_c << B$ no great error is introduced by considering W (fd) = constant = W (0) which, in many cases, simplifies the following calculations considerably, see also Fig. A. 4. This means that only the very low frequency components play an important role in the determination of the standard deviation of the energy fluctuations.
Four types of linear averaging circuits are of interest in the determination of the effective averaging (sampling) time of the Level Recorder Type 2305.

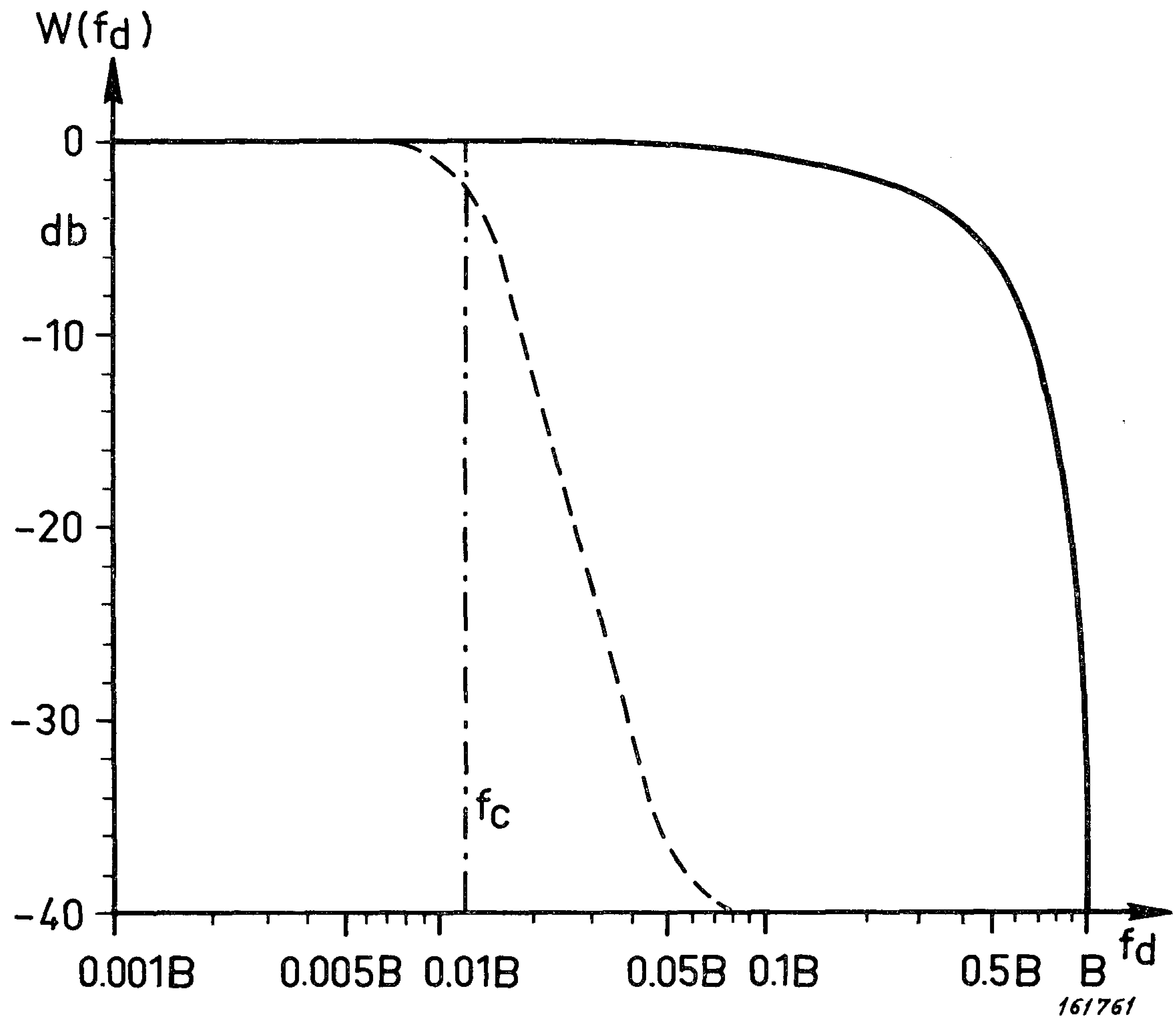


Fig. A.4. The same spectrum as shown in Fig. A.3. Also the frequency response of a relatively sharp averaging filter with cut off frequency f_c is shown.

1. Theoretically correct sampling over periods of time T.
2. An ordinary R-C type averaging network.
3. An R-L-C type filter with low Q-value.
4. A critically damped R-L-C type filter.

1. *Theoretically correct sampling with the sampling time T.*
It can be shown that the frequency spectrum of a rectangular unit pulse of duration T is

$$G_1(f) = \frac{\sin \pi fT}{\pi f}$$

It can also be shown that this spectrum can be considered the "filter characteristic" of the sampling (see Fig. A. 5). The standard deviation (σ_1) of the energy fluctuations of the sampled noise signal is therefore: —

$$\sigma_1^2 = \int_0^\infty W(0)\ G_1^2(f)\ df = \int_0^\infty 2\ \alpha^2\ w^2\ (f)\ B\ \frac{\sin^2(\pi fT)}{\pi^2 f^2}\ df = \alpha^2\ w^2\ (f)\ B\ T$$

whereby:

$$\sigma_1 = \alpha\, w(f)\ \sqrt{BT}$$

The mean energy, $\overline{E}$, per unit time measured by means of the same squaring device ($Y = \alpha x^2$) is $\alpha\, w(f)\ B$, and the total energy, $\overline{E}_T$, after a time T is $\alpha\, w(f)\ B\ T$.

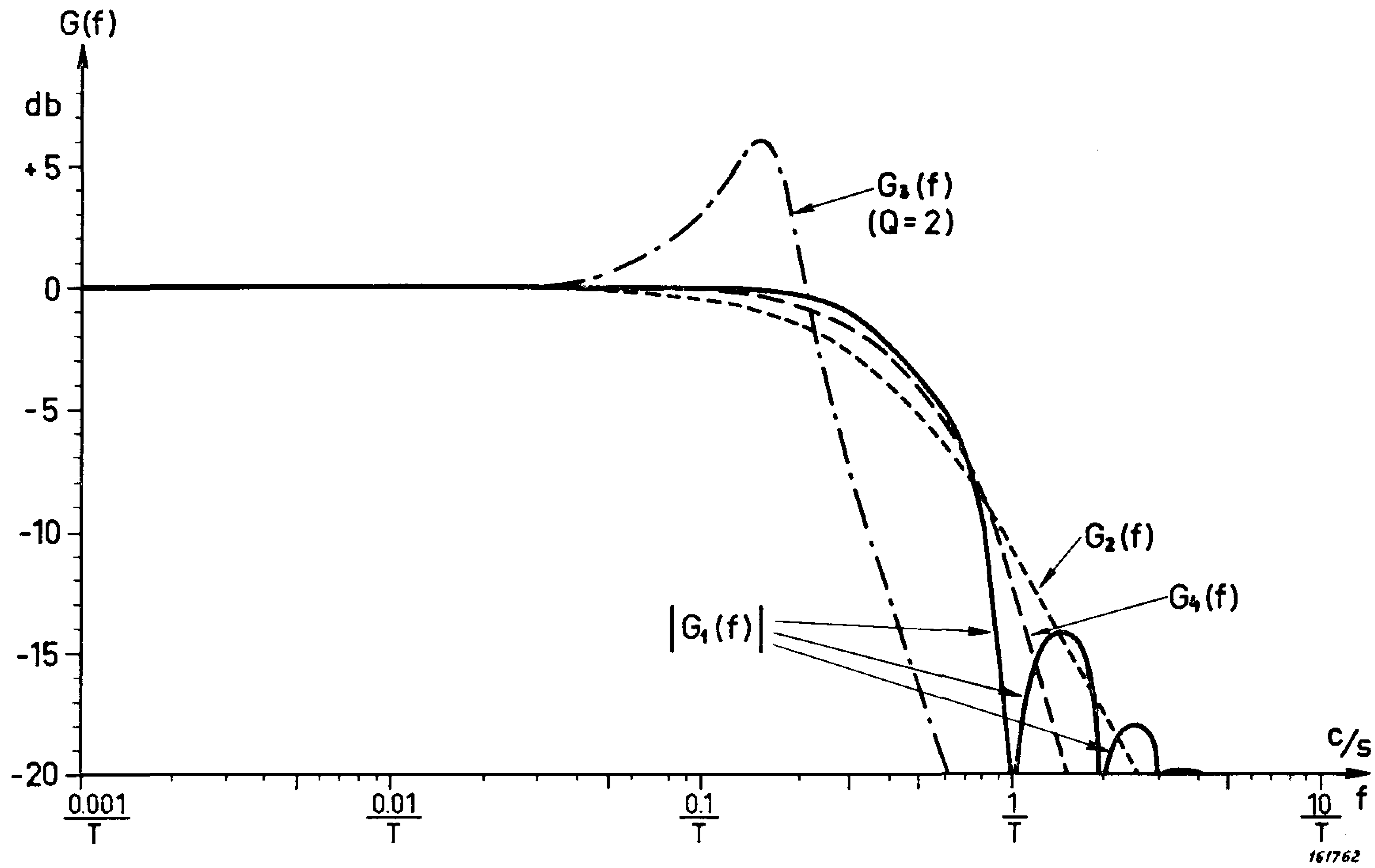


Fig. A.5. Typical frequency response curves of averaging filters which give the same effective averaging time T.

$G_1(f)$: Absolute response of theoretically ideal sampling.

$G_2(f)$: R-C type averaging filter.

$G_3(f)$: R-L-C type averaging filter with $Q = 2$.

$G_4(f)$: Critically damped R-L-C type averaging filter.

The standard deviation of the energy fluctuations is thus: —

$$\sigma_1 = \overline{E}_T \times \frac{\sqrt{BT}}{BT} = \frac{\overline{E}_T}{\sqrt{BT}}$$

or:

$$\underline{\frac{\sigma_1}{\overline{E}_T} = \frac{1}{\sqrt{BT}}}$$

2. *Ordinary R-C Type Averaging Network.*

The frequency response $G_2(f)$ of this type of network is: —

$$G_2(f) = \frac{1}{\sqrt{\left(\frac{f}{f_o}\right)^2 + 1}}$$

where $f_o = \frac{1}{2\pi RC}$ (3 db upper limiting frequency of the R-C filter)

Thus:

$$\sigma_2^2 = \int_0^\infty W(0)\, G_2^2(f)\, df = \int_0^\infty \frac{2\,\alpha^2\, w^2(f)\, B}{\left(\frac{f}{f_o}\right)^2 + 1} = \alpha^2\, w^2(f)\, B\, \pi\, f_o$$

and $\sigma_2 = \alpha\, w(f)\, \sqrt{B\, \pi\, f_o}$

The mean energy per unit time is the same as above, whereby the standard deviation of the energy fluctuations is: —

$$\sigma_2 = \overline{E}\, \frac{\sqrt{\pi\, f_o\, B}}{B} \quad \text{or} \quad \frac{\sigma_2}{\overline{E}} = \sqrt{\frac{\pi\, f_o}{B}}$$

3. *R-L-C Type Filter.*
The frequency response $G_3(f)$ of such a network is given by:

$$G_3(f) = \frac{1}{\sqrt{\frac{1}{Q^2}\left(\frac{f}{f_r}\right)^2 + \left[1 - \left(\frac{f}{f_r}\right)^2\right]^2}}$$

where $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ = resonance frequency of the circuit.

Then:

$$\sigma_3^2 = \int_0^\infty W(0)\, G_3^2(f)\, df = \int_0^\infty \frac{2\,\alpha^2\, w^2(f)\, B}{\frac{1}{Q^2}\left(\frac{f}{f_o}\right)^2 + \left[1 - \left(\frac{f}{f_r}\right)^2\right]^2}\, df = \alpha^2\, w^2(f)\, B\, \pi\, f_r\, Q$$

and $\sigma_3 = \alpha\, w(f)\, \sqrt{\pi\, f_r\, Q\, B}$

Again taking the relative standard deviation of the energy fluctuations:

$$\frac{\sigma_3}{\overline{E}} = \sqrt{\frac{\pi\, f_r\, Q}{B}}$$

4. *Critically Damped R-L-C Type Filter.*
This case is actually the same as considered under (3) above with $Q = \frac{1}{2}$. The resonance frequency, f_r, will now be the 6 db upper limiting frequency of the filter:

$$\frac{\sigma_4}{\overline{E}} \sqrt{\frac{\pi\, f_r}{2\, B}}$$

Selected References:

1. S. O. Rice: "Mathematical Analysis of Random Noise", Bell System Techn. Journal 23 (1944), 24 (1945), also contained in N. Wax: "Selected Papers on Noise and Stochastic Processes", Dover Publications, Inc., New York 1954.

2. J. S. Bendat: "Principles and Applications of Random Noise Theory", John Wiley & Sons, Inc., 1958.

3. S. H. Crandall: "Notes for the M. I. T. Special Summer Program on Random Vibration". The Technology Press of the Massachusetts Institute of Technology, Cambridge 39, Massachusetts, 1958.

4. R. R. Bennett and A. S. Fulton: "The Generation and Measurement of Low Frequency Random Noise", Journal of Applied Physics, Volume 22, Number 9, 1951.

5. C. T. Morrow: "Averaging Time and Data-Reduction Time for Random Vibration Spectra" (I & II), J.A.S.A., Volume 30, Numbers 5 and 6, 1958.

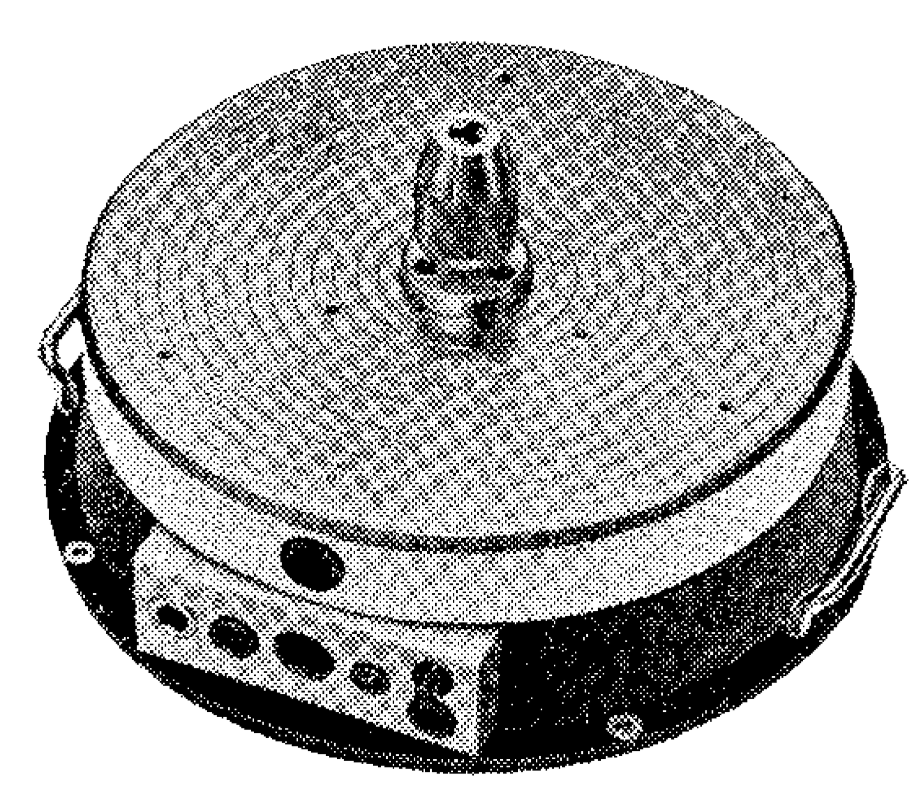
6. J. T. Broch and C. G. Wahrman: "R.M.S. Recording of Narrow Band Noise with the Level Recorder Type 2305", B & K Technical Review No. 4-1960.

News from the Factory.

Turntable Type 3921.

In continuation of our recently released Level Recorder Type 2305 an entirely new Turntable Type 3921 has been designed. By combining the new Level Recorder and this Turntable a complete polar recorder is derived, thus enabling directional characteristics of various objects to be automatically recorded.

The Turntable is built in two main elements, the table base and a rotatable horizontal table plate. The latter has a diameter of approximately 350 mm and is capable of taking a centered load of 100 kg. In the center of the table plate is a chuck in which mounting rods of diameter up to 16 mm can be clamped. When a plain table plate is desired, the chuck can be readily removed. The table base and table plate are mechanically coupled via a friction clutch, which enables any clamped test specimen on the table plate to be manually orientated before measurement in any desired direction in relation to a receiving or transmitting equipment situated away from the table.



The table plate is driven from a built-in self-starting synchronous motor, which gives the table an r.p.m. of 0.75 i.e. one revolution takes 80 sec. When the Level Recorder Type 2305 and Turntable are powered from the same line voltage (frequency) a polar paper inserted in the Recorder will run synchronously with the table, provided the "Paper Speed" on the Recorder is set to "10" mm/sec. As no mechanical shaft is necessary for driving of the Turntable the Recorder can be placed at any distance from the Table during measurement, this being very convenient in many applications.

One complete revolution of the table plate, and with it the polar paper, can be started from the Turntable or the Recorder. In addition, the commencement of the run can be remotely controlled. After exactly one revolution the Table and Recorder automatically stop. To derive the simultaneous start and stop function a three-cored cable has to be connected from the Level Recorder to the Turntable. Two types of connection can be made from the base of the Turntable via slip-rings to the specimen on the table plate.

When taking polar recordings the special polar recording paper QP 5102 should be used in the Recorder. This recording paper is preprinted in degrees and db, and is intended for ink writing. The diameter is approximately 260 mm.

The drive system of the Turntable can be delivered (on request) for two different line frequencies, 50 c/s or 60 c/s, and is intended for the same line voltage as the Level Recorder Type 2305.

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